Ontological Logs, Higher Categories

and a path to programmatic representation

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Part 1: Intuitions and Definitions

- Ologs
- $\mathfrak{C}at \implies (\mathfrak{C}at \downarrow \mathfrak{C}) \implies sm(\mathfrak{C})$
- Colimits / Coverings
- Anisotropy / monoidial $\$$
- ꩜ / $\oplus$
- Site / Ontological Generation

Part 2: $sm : \mathfrak{C}at \to \mathfrak{C}at$, 2-categories

- $\mathfrak{C}at$ as a category, $sm, Fun : \mathfrak{C}at \to \mathfrak{C}at$
- $\Delta : Id_{\mathfrak{C}at} \to Fun$
- $colim : Fun(\mathfrak{C}) \to \mathfrak{C}$
- $\eta : Id \to \Delta \circ colim$ as initial in $\mathscr{L}_{fun,\mathfrak{C}}$

Part 3: Higher Categories: Simplicial sets, Python Ologs

- $\mathfrak{C}at$ as a 2-category
- Simplicial Sets
- Functorality as Naturality
- Pythonic Simplicial Sets
- Face and Degeneracy as ontological expansions

Ologs

Ontological Log

An **Ontological Log** is a labeled category.

- I.e. labeled objects and labeled morphisms
- An ontology represents concepts and their relations via category theory

Ontological Expansions

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This is formalized by an **ontological expansion**

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Submorphism

A **submorphism** $\mathcal{F} : J \to J'$ is a set of maps between the images of J and J'

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an **Ontological Expansion** is a functor $O : \mathfrak{C} \to sm(\mathfrak{D})$

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in this case, for an ontological expansion $O : \mathfrak{C} \to sm(\mathfrak{D})$ we then have a covering $\{\eta_s : O(c)(s) \to colim(O(c))\}$

Colimits and Coverings, First OG assumption

If we make the further assumption that:

$$O : \mathfrak{C} \rightarrow sm(\mathfrak{C}) \text{ and } colim(O(c)) = c$$

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This assertion is the first condition of an **Ontological Generator**

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Goal

To organize different expansions and how they relate

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(cellular anatomy is much more data than macroscopic body parts)

- We also want to relate two ontological expansions

So our proto-definition of an ontological generator is now a functor

$$OG : \$ \to sm(\mathfrak{C})^{\mathfrak{C}}$$

Operations on Ontological Expansions: 9

- recall that $sm : \mathfrak{C}at \to \mathfrak{C}at$ is a functor
- given $O : \mathfrak{C} \to sm(\mathfrak{C})$, $sm(O) : sm(\mathfrak{C}) \to sm(sm(\mathfrak{C}))$

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given two ontological expansions $O, O' : \mathfrak{C} \to sm(\mathfrak{C})$ we can create a composable chain:

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This composition yeilds the **spiral product**

$$O' \mathbin{\unicode{x0669}} O = colim \circ sm(O') \circ O$$

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- Finally collect ontologies into one large on that "looks like"

$$\bigcup_{c' \in O(c)} O'(c')$$

(careful, in general the spiral product isn't just an ordinary union)

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We want to actually measure data about the objects of $\mathfrak{C}$. The site assumption allows us to formalize data in terms of sheaves $\mathscr{F} : \mathfrak{C} \to \mathfrak{D}$.

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Moreover given an ontological expansion $OG_s(c)$, we can use the sheaf condition to "glue together" data from $c' \in OG_s(c)$ to c-data

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3)$Cov = \{OG(s)(c) \to c\}_{s \in \$, c \in \mathfrak{C}}$ makes $\mathfrak{C}$ into a site

Part 2: $sm : \mathfrak{C}at \to \mathfrak{C}at$ and colim

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- Conjecture: $\eta : Id \to \Delta \circ colim$ is initial in $\mathscr{L}_{sm,\mathfrak{C}}$

$(\mathfrak{C}at \downarrow \mathfrak{C})$

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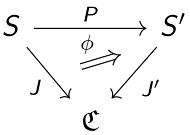
P is a functor and ϕ a natural transformation as in the diagram:



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That is, $fun(F)$ is a functor:

- for $J : S \to \mathfrak{C}$, $fun(F)(J) = F \circ J : S \to \mathfrak{D}$
- for $(P, \phi : J \to J' \circ P)$, $fun(F)(P, \phi) = (P, F(\phi))$

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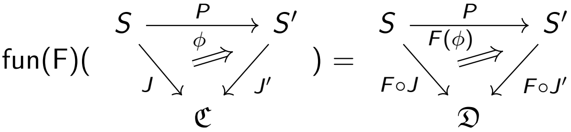

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Consider the identity functor $Id_{\mathfrak{C}at}$. Both fun and Id are endofunctors of the category $\mathfrak{C}at$

- Δ is a natural transformation $\Delta : Id_{\mathfrak{C}at} \to fun$.

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Colimit as a natural transformation

- Let $L : fun(\mathfrak{C}) \to \mathfrak{C}$ a functor
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The canonical morphisms give us a natural $\eta : Id \to \Delta \circ colim$

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let $\mathscr{L}_{fun,\mathfrak{C}}$ be the category such that:

- objects are pairs $(L, l : Id_{fun(\mathfrak{C})} \to \Delta_C \circ L)$
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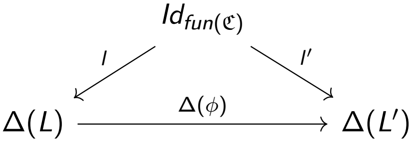

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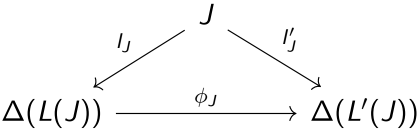

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$$\begin{array}{ccc} & J & \\ {\scriptstyle l_J}\swarrow & & \searrow{\scriptstyle l'_J} \\ \Delta(L(J)) & \xrightarrow{\phi_J} & \Delta(L'(J)) \end{array}$$

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$\phi_J : L(J) \to L'(J)$ such that for all $s \in S$

$$\begin{array}{ccc} & J(s) & \\ {\scriptstyle l_{J,s}}\swarrow & & \searrow{\scriptstyle l'_{J,s}} \\ L(J) & \xrightarrow{\phi_J} & L'(J) \end{array}$$

colim is initial in $\mathscr{L}_{fun,\mathfrak{C}}$

If $\mathfrak{C}$ is cocomplete, the universal property of the colimit is exactly the statement that, for all L,J, there is a unique $\phi_J : colim(J) \to L(J)$ such that

$$\begin{array}{ccc} & J(s) & \\ {\scriptstyle \eta_{J,s}} \swarrow & & \searrow {\scriptstyle l_{J,s}} \\ colim(J) & \xrightarrow{\Delta(\phi)} & L(J) \end{array}$$

that is, colim is initial in the category $\mathscr{L}_{fun,\mathfrak{C}}$

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that is, i forgets that ϕ is a natural transformation, and instead regards it just as a collection of maps $\phi_s : J(s) \to J'(P(s))$, i.e. a submorphism.

Conjecture: i^* is faithfull and essentially surjective

Lift i to a functor $i^* : \mathfrak{C}^{sm(\mathfrak{C})} \to \mathfrak{C}^{fun(\mathfrak{C})}$:

$$i^*(L)(J) = L \circ i(J)$$

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if this is true then $\mathscr{L}_{sm,\mathfrak{C}}$ has an initial object, which will be the colimit $sm(\mathfrak{C}) \to \mathfrak{C}$ we are looking for, but this is still left to prove.

Part 3: Towards higher ontologies; programmatic representations

- $\mathfrak{C}at$ and the 2-categorical secret
- simplicial sets
- higher ontologies
- faces and degeneracies as ontological expansions
- programming higher ontologies

$\mathfrak{C}at$ as a 2-category

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2-morphisms are commutative diagrams:

$$\begin{array}{ccc} \mathfrak{C} & \xrightarrow{P} & \mathfrak{D} \\ \downarrow F & \overset{\eta}{\nearrow\!\!\!\nearrow} & \downarrow F' \\ \mathfrak{C} & \xrightarrow{Q} & \mathfrak{D}' \end{array}$$

where $\eta : Q \circ F \to F' \circ P$ is a natural transformation

Simplecies in $\mathfrak{Cat}$

The diagram $$\begin{array}{ccc} \mathfrak{C} & \xrightarrow{P} & \mathfrak{D} \\ {\scriptstyle F}\downarrow & \overset{\eta}{\nearrow\!\!\!\nearrow} & \downarrow{\scriptstyle F'} \\ \mathfrak{C}' & \xrightarrow{Q} & \mathfrak{D}' \end{array}$$

Can actually be described entirely by a "2-simplex":

$$\begin{array}{ccc} \mathfrak{C} & & \\ {\scriptstyle F}\downarrow & \overset{\eta}{\nearrow\!\!\!\nearrow} & \searrow{\scriptstyle F' \circ P} \\ \mathfrak{C}' & \xrightarrow[Q]{} & \mathfrak{D}' \end{array}$$

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Just as two 0-simplecies(categories) can have multiple 1-simplecies(functors) between them
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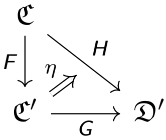

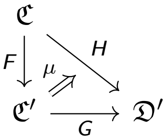

face and degeneracy

given a 2-simplex $\sigma^2 = \begin{array}{ccc} \mathfrak{C} & & \\ {\scriptstyle F}\downarrow & \overset{\eta}{\nearrow\!\!\!\nearrow} & \searrow{\scriptstyle H} \\ \mathfrak{C}' & \xrightarrow[G]{} & \mathfrak{D}' \end{array}$

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we can extract 3 bits of information, namely the faces:
$\mathit{face}_0(\sigma^2) = H$, $\mathit{face}_1(\sigma^2) = F$, $\mathit{face}_2(\sigma^2) = G$

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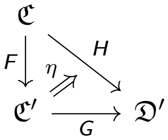

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On the other hand, given a 1-simplex: $\sigma^1 = F : \mathfrak{C} \to \mathfrak{D}$, we can get two "degenerate" 2-simplecies:

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$$d_0(\sigma^1) = \begin{tikzcd} \mathfrak{C} \arrow[d, "Id_{\mathfrak{C}}"'] \arrow[dr, "F"] & \\ \mathfrak{C} \arrow[r, "F"'] \arrow[ur, Rightarrow, "Id"] & \mathfrak{D} \end{tikzcd} \quad \text{and } d_1(\sigma^1) = \begin{tikzcd} \mathfrak{C} \arrow[d, "F"'] \arrow[dr, "F"] & \\ \mathfrak{D} \arrow[r, "Id_{\mathfrak{D}}"'] \arrow[ur, Rightarrow, "Id"] & \mathfrak{D} \end{tikzcd}$$

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if Σ_n is the set of n-simplecies, the faces and degeneracies are, in general, functions:

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of course, in more general situations, this chain continues as:

$$\Sigma_{n-1} \underset{\{f_i^n\}_{0\leq i\leq n}}{\longleftarrow} \Sigma_n \overset{\{d_i^n\}_{0\leq i\leq n}}{\longrightarrow} \Sigma_{n+1}$$

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A morphism of simplicial sets, is then just a natural transformation $\eta : \Sigma \to \Sigma'$

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i.e. a natural transformation between 1-categories (as simplicial sets) is actually just a functor.

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The theory of Simplicial Sets is pretty well developed (almost as well as category theory) and so sSet gives a great starting point for capturing the intuition behind what a higher ontology should be

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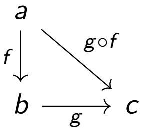

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A simplicial set is then just a collection of simplex objects containing its degeneracies and faces

Pythonic Functors

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you can view my working code at https://github.com/nopounch/golog

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The functorality here isn't apparent, and may necessitate working with something close to but not exactly simplicial sets.

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